



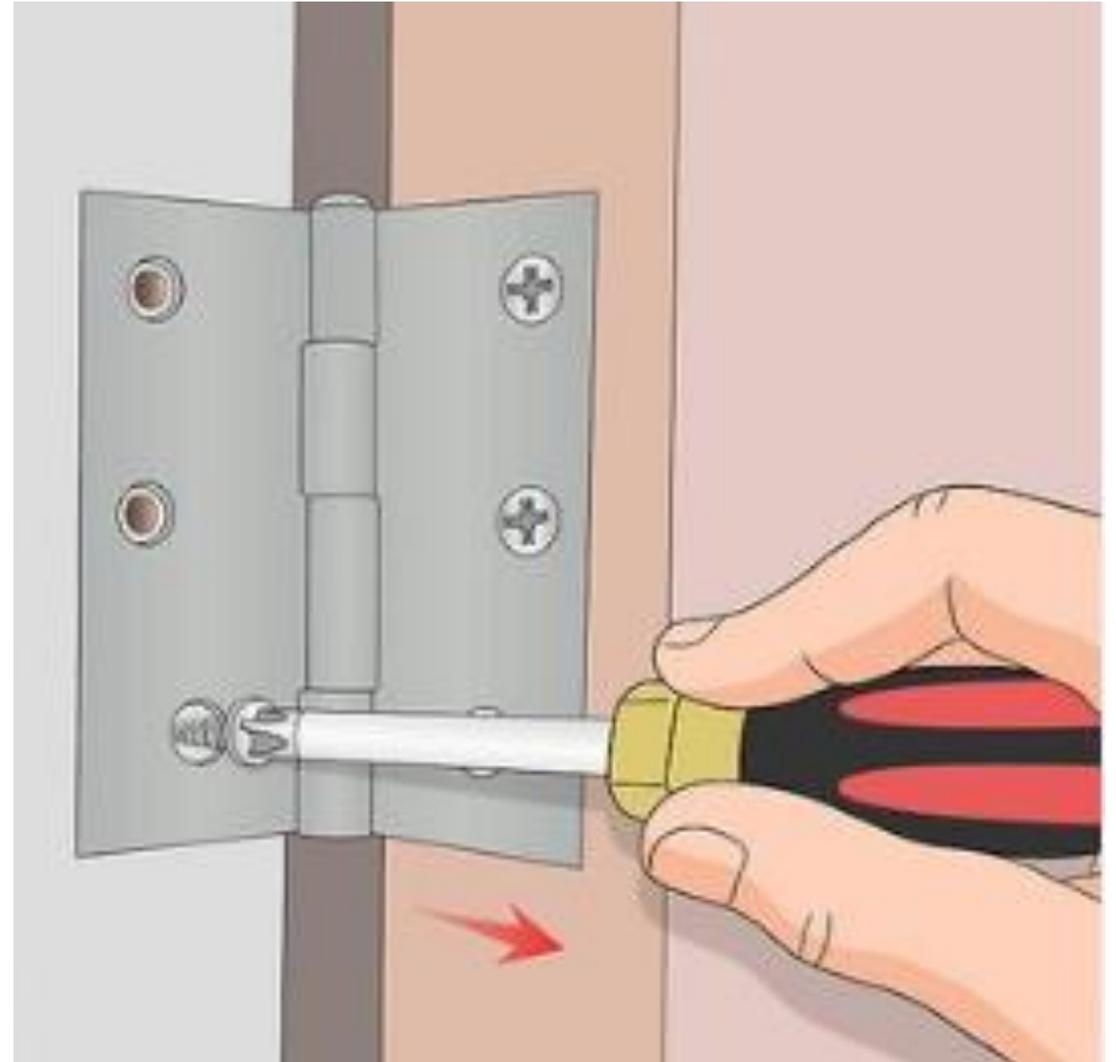
# CROSS PRODUCT or Vector product

## VECTORS: LESSON SIX

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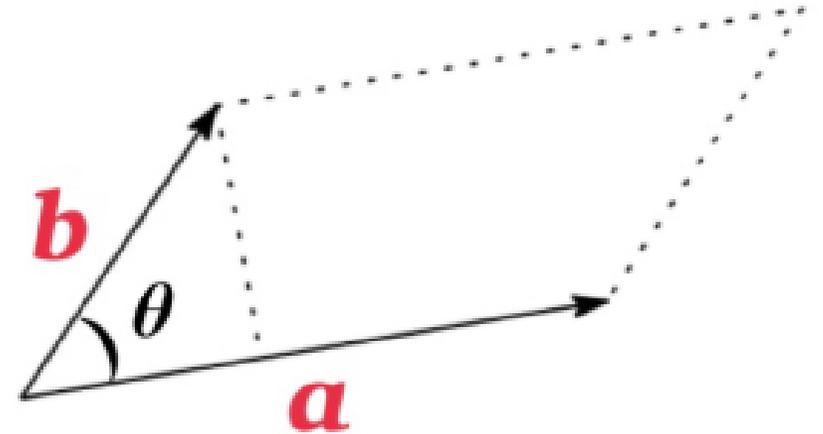




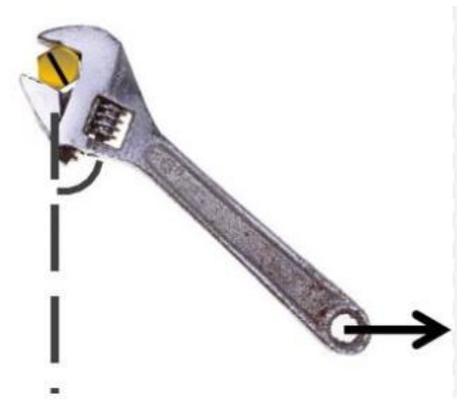
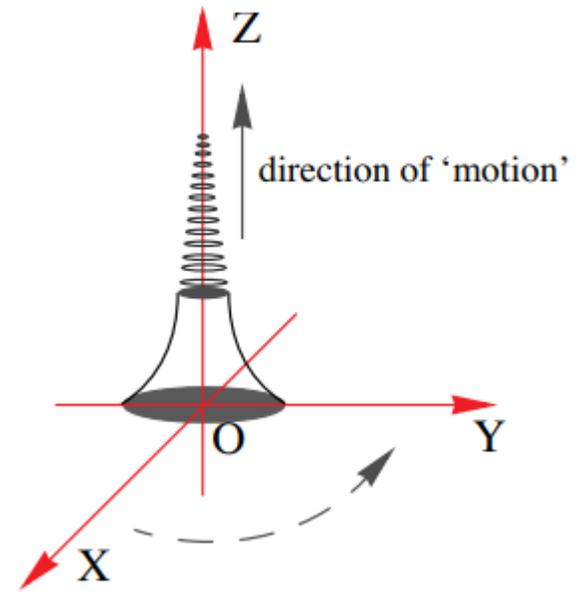
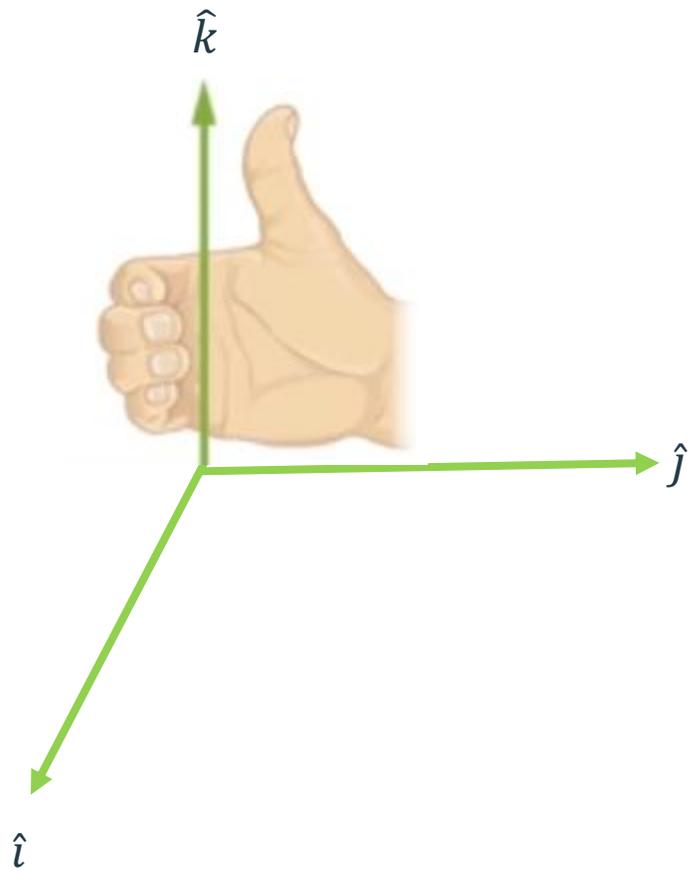
# Definition

- This is an operation on two or more vectors to form another vector. The cross product of two vectors  $\mathbf{a}$  and  $\mathbf{b}$  is written as  $\mathbf{a} \times \mathbf{b}$  and is defined as

$$\vec{\mathbf{a}} \times \vec{\mathbf{b}} = |\vec{\mathbf{a}}| |\vec{\mathbf{b}}| \sin\theta \hat{\mathbf{n}}$$



and  $\theta$  is the angle between  $\mathbf{a}$  and  $\mathbf{b}$  and  $\hat{\mathbf{n}}$  is a unit vector perpendicular to both  $\mathbf{a}$  and  $\mathbf{b}$ ,



## ACTIVITY

- Given that  $a = 2i + 3j - 4k$  and  $b = 3i - 5j + k$ . Find  $a \times b$

# ACTIVITY

A mechanical engineer is installing a rotating arm on an industrial machine. Two forces act on the arm at the same point, and their directions are represented by the vectors:  $\vec{a} = -3i + 2j - 3k$  and  $\vec{b} = -i - 3j + k$  the machine to rotate correctly and safely, the engineer needs to determine the direction in which the arm will rotate. This direction must be perpendicular to both forces acting on the arm.

Task

- Using vector methods, determine the direction of rotation of the arm.

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# APPLICATIONS

## ◆ 1. Finding the Area of a Parallelogram

The magnitude of the cross product gives the area of a parallelogram formed by two vectors.

$$\text{Area} = |\vec{a} \times \vec{b}|$$

### 📌 Application:

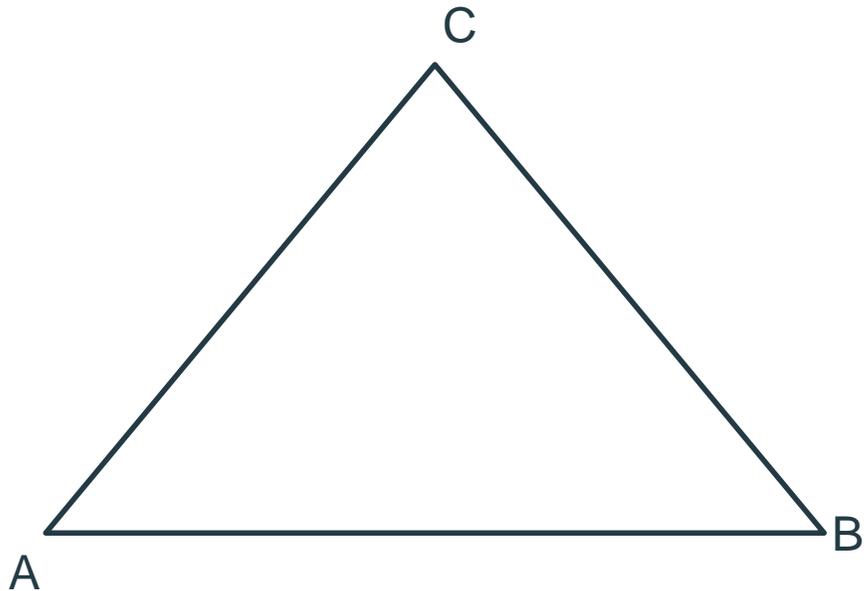
- Finding land area
- Surveying
- Engineering designs

➡ If the area of a triangle is needed:

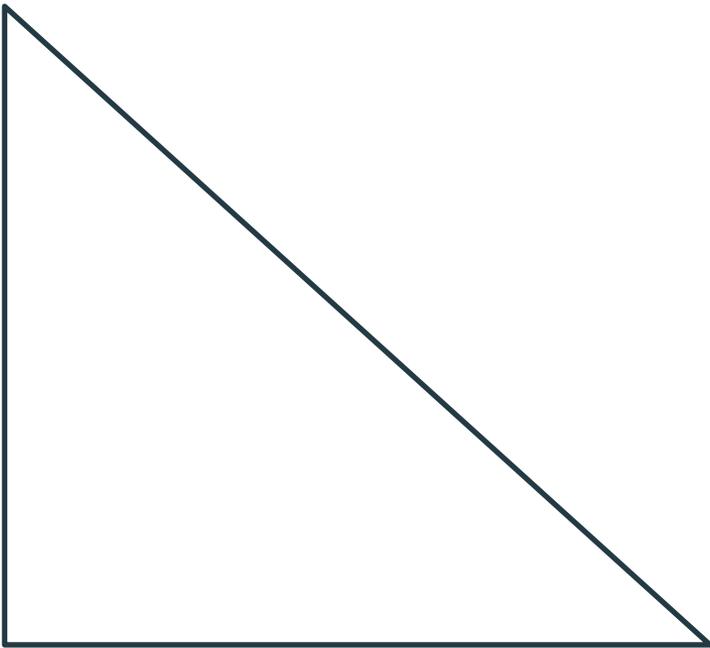
$$\text{Area} = \frac{1}{2} |\vec{a} \times \vec{b}|$$

# Applications

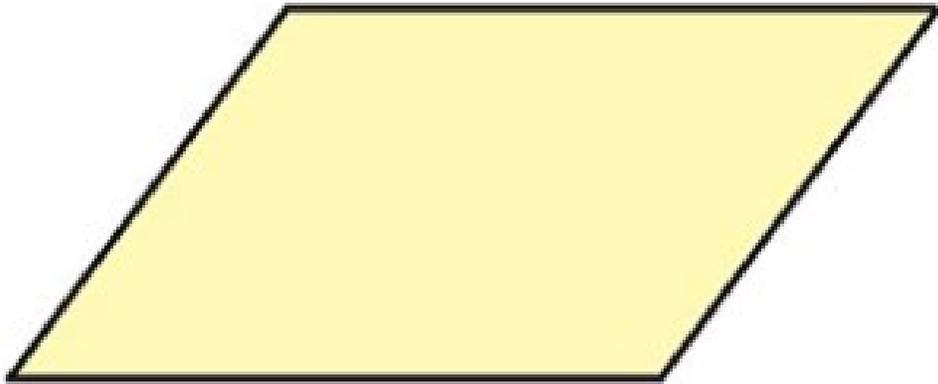
- To show that given vertices are for a triangle



To show that a given triangle is right angled triangle

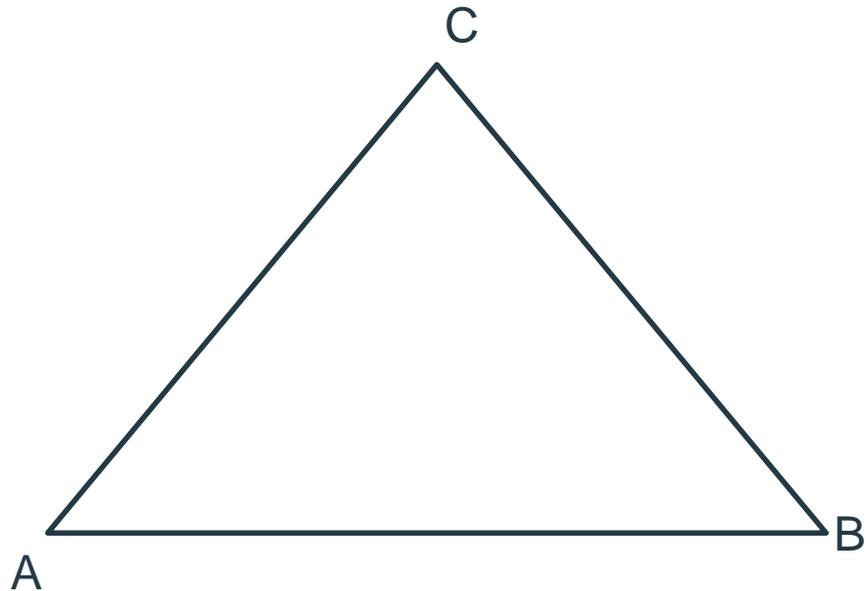


# AREA OF A PARALLELOGRAM



# More Applications

- To Find the area of triangle



# ACTIVITY

Three points ABC have coordinates  $A(3,3,1)$  , $B(8,7,4)$  and  $C(11,4,5)$   
.Show that the points are vertices of a triangle. Hence find the area of the triangle ABC

# ACTIVITY

A land surveyor is mapping three locations on a construction site. The positions of the three points are given by: P (13, -2, 0) Q (7, 1, -3) R (2, -1, 5). The surveyor wants to determine the shape formed by joining these three points and to know whether the layout satisfies safety requirements for construction.

## Tasks

- a) Using vector methods, determine whether the points P, Q, and R form a triangle.
- b) If a triangle is formed, determine whether it is right-angled.
- c) Find the area of the triangular region formed by the three points.

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# EXERCISE

A surveyor is mapping a triangular piece of land for construction. From the same corner point  $O$ , two boundary lines of the land extend in the directions:  $\vec{a}=3\vec{i}+4\vec{j}$  and  $\vec{b}=5\vec{i}-2\vec{j}$ . The surveyor needs to determine how big the land is in order to estimate construction costs.

Task. Help the surveyor to determine the size of the land.

A surveyor is mapping a piece of land that forms a parallelogram ABCD. Three of the vertices of the land are known and have the following coordinates: B (1, -2), C (8, -5), D (5, 0). The fourth point A is not marked on the map, but the surveyor knows that the shape formed is a parallelogram.

Task

Determine the coordinates of point A. Hence, determine the size of the land enclosed by the parallelogram.

A designer is modelling a structure using a 3-dimensional coordinate system. Three points on the structure are located at: A (1, 1, 2), B (2, 1, 5), C (4, 0, 1). The fourth point D is positioned such that the four points form a parallelogram. Later, another point E is placed on the line joining B and D, in such a way that B lies midway between E and D. A final point F is then positioned so that ABEF also forms a parallelogram.

### Tasks

- a) Determine the coordinates of point D and find the area of the figure ABCD.
- b) Find the coordinates of point E.
- c) Determine the coordinates of point F.
- d) Verify that B is the midpoint of FC.
- e) Hence, explain why ADBF is also a parallelogram.